

WORDS TO KNOW

greatest common factor

least common multiple

Lesson 10

FIND COMMON FACTORS AND
COMMON MULTIPLES NY-6.NS.4

INTRODUCTION

Real-World Connection

Anika and her family are waiting at the airport for a shuttle to take them to their car. They need two shuttles, but both shuttles are leaving as they walk to the shuttle area. One of the shuttles returns to the airport every 9 minutes. The other shuttle returns every 12 minutes. How long will it be before the two shuttles are at the airport at the same time again? Let's practice the skills in the **Guided Instruction** and **Independent Practice** and, at the end of the lesson, see how long Anika and her family have to wait!



What I Am Going to Learn

- How to find the greatest common factor of two numbers
- How to find the least common multiple of two numbers
- How to use the Distributive Property to express a sum of two numbers with a common factor, as a multiple of a sum of two numbers with no common factors

What I May Already Know

- I know how to multiply and divide numbers.
- I know how to find all factor pairs for any number up to 100.
- I know if a number up to 100 is a multiple of a single-digit number.

Vocabulary in Action

There are a few ways to see how numbers are related.

- The **greatest common factor** (GCF) of a set of numbers is the greatest factor they have in common. For 24 and 36, 12 is the GCF.
- The **least common multiple** (LCM) of a set of numbers is the least multiple they have in common. For 24 and 36, 72 is the LCM.

The Distributive Property states: $a(b + c) = ab + ac$. This can be used with factors.

- b and c have a as a common factor. If it is the GCF, b and c will have no other common factors: $24 + 36 = 12(2 + 3) = 12 \times 2 + 12 \times 3$.

EXAMPLE

Find the greatest common factor of 12 and 18.

Factors of 12: 1, 2, 3, 4, 6, 12 (1×12 , 2×6 , 3×4)

Factors of 18: 1, 2, 3, 6, 9, 18 (1×18 , 2×9 , 3×6)

There are several common factors: 1, 2, 3, and 6.

As 6 is the greatest of the common factors, the GCF of 12 and 18 is 6.

If there were 12 boys and 18 girls at a summer camp, and they wanted to make as many teams as possible, each with the same number of boys and girls, there would be 6 teams with 2 boys and 3 girls: $6(2 + 3)$.

TIPS AND HINTS

List factors systematically: will 1 work? or 2? or 3? and so on. For each factor that works, list its partner. Stop when factors repeat.

Use the Distributive Property with greatest common factor.

EXAMPLE

Rewrite $16 + 24$ using the Distributive Property and GCF.

List the factors of each number:

16: 1, 2, 4, **8**, 16

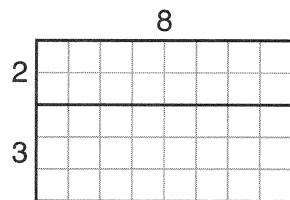
24: 1, 2, 3, 4, 6, **8**, 12, 24

The greatest common factor is 8.

Use the Distributive Property.

$$16 + 24 = 8(2 + 3)$$

$$8(2 + 3) = 8 \times 2 + 8 \times 3$$



You have used the Distributive Property in a similar way to break up number facts such as 8×5 into $8 \times 2 + 8 \times 3 = 8(2 + 3)$.

TIPS AND HINTS

Sometimes, you might look at two numbers and know the GCF right away: "Oh, 16 is 8×2 and 24 is 8×3 , so 8 is the GCF." If so, there is no need to list the factors. Just double check that there are not any more common factors.

Any two numbers have an infinite number of common multiples. The LCM is the *least* common multiple.

THINK ABOUT IT

The product of 2 numbers will always be a common multiple, but not necessarily the least common. However, if the numbers have no factors in common, such as 4 and 7, the LCM is the product: 28.

EXAMPLE

Find the least common multiple of 4 and 6.

List the first few multiples of each number:

4: 4, 8, **12**, 16

6: 6, **12**, 18

You can stop when you find a common multiple. 12 is the LCM.

Suppose you are planning a picnic and you buy hot dogs in packs of 4 and buns in packs of 6. What is the fewest number of packs you should buy to get the same number of hot dogs and buns?

You would buy 3 packs of hot dogs ($4 \times 3 = 12$)

and 2 packs of buns ($6 \times 2 = 12$).

GUIDED INSTRUCTION

TIPS AND HINTS

If you see a common factor, start there. But, look at the remaining factors to see if those are common.

1. Find the greatest common factor of 48 and 72.

Step One List the factors of 48 and 72.

48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Step Two What is the greatest common factor?

48 and 72 are both multiples of 24:

$$24 \times 2 = 48$$

$24 \times 3 = 72$, so is the GCF.

Another Way You may have seen a common factor of 12 right away.

$$48 = 12 \times 4$$

$$72 = 12 \times 6$$

4 and 6 share a common factor of 2.

$$48 = 12 \times 2 \times 2$$

$$72 = 12 \times 2 \times 3$$

$$48 = (12 \times 2) \times 2 = 24 \times 2$$

$$72 = (12 \times 2) \times 3 = 24 \times 3$$

You can see the factor of 24, and 2 and 3 have no common factor. So, 24 is the GCF.

2. Find the least common multiple of 4 and 9.

Step One List the multiples of 4 and 9.

4: 4, 8, 12, 16, 20, 24, 28, 32, 36

9: 9, 18, 27, 36

Step Two What is the least common multiple?

The LCM is:

Another Way You may have noticed that 4 and 9 have no common factors, so the first common multiple is 4×9 .

4: 1, 2, 4

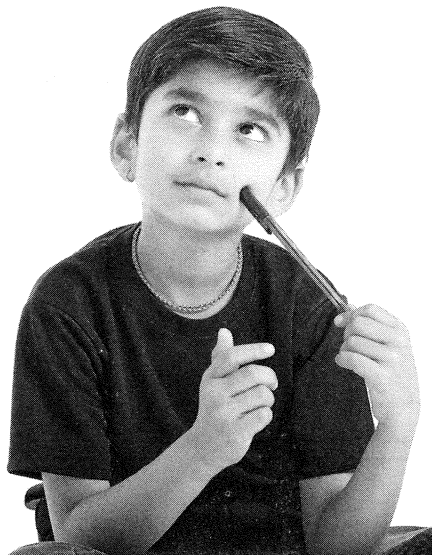
9: 1, 3, 9

3. Which statement is true?

- (A) The numbers 8 and 12 have three common factors.
- (B) The greatest common factor of 12 and 28 is 2.
- (C) The greatest common factor of 35 and 100 is 7.
- (D) Using the Distributive Property, $12 + 15$ can be rewritten as $3(3 + 5)$.

◀ **TIPS AND HINTS**

Think about what the pairs of numbers have in common. Where do you see them in a multiplication table?



Learning Together

Work with a partner. Write three new problems that can be solved by finding the greatest common factor. Think about situations where you need to make equal groups of two different items. Write each final problem neatly on a note card and display all the problems on the bulletin board. Solve all the problems as a class. When the class solves your problems, be prepared to tell if the answer is correct or not correct and why.



How Am I Doing?

What questions do you have?

How can you find the LCM and GCF of two numbers?

What is an example where greatest common factor or least common multiple would be used in everyday life?

Circle the sign that shows how you are doing with the skill.



I am stuck.



I almost have it.



I understand the skill.

INDEPENDENT PRACTICE 1

1 Jose set his locker combination to be all the factors of 14. Which is his locker combination?

- A 1, 2, 3, 4, 6
- B 1, 2, 7
- C 1, 2, 7, 14
- D 14, 28, 42, 56

2 For which pair of numbers is 24 the least common multiple?

- A 2 and 12
- B 3 and 4
- C 4 and 6
- D 6 and 8

3 Which of the following pairs of numbers has 12 as the greatest common factor?

- A 24 and 48
- B 12 and 18
- C 6 and 4
- D 36 and 48

◀ TIPS AND HINTS

Numbers have a limited (finite) number of factors but an infinite number of multiples.

◀ THINK ABOUT IT

How are “common multiple” and “least common multiple” different?

◀ TIPS AND HINTS

Factors will always be less than or equal to the number. Multiples will always be greater than or equal to the number.

4

Is 40 the least common multiple of 4 and 10?

Explain your answer.

◀ SKETCH IT

Write the numbers 1-40 twice in side-by-side columns. In the left column, circle the multiples of 4. In the right column, circle the multiples of 10. Draw arrows showing that 40 is a common multiple. Draw an arrow to show each other common multiple.



INDEPENDENT PRACTICE 2

1

Which statement is correct?

- A The least common multiple of 6 and 7 is 21.
- B The least common multiple of 9 and 5 is 45.
- C The least common multiple of 3 and 6 is 9.
- D The least common multiple of 11 and 12 is 1.

2

Which equation is true and uses the GCF of 12 and 30?

- A $12 + 30 = 2(6 + 15)$
- B $12 + 30 = 3(4 + 10)$
- C $12 + 30 = 6(2 + 5)$
- D $12 + 30 = 6(2 + 7)$

3

Which pair of numbers has the same least common multiple as 3 and 10?

- | | |
|-----------|------------|
| A 2 and 5 | C 5 and 10 |
| B 5 and 6 | D 8 and 3 |

4

Which of the following expressions correctly shows the use of the distributive property with the greatest common factor for the addition of $36 + 54$?

- A $2(18 + 27)$
- B $3(12 + 18)$
- C $18(2 + 3)$
- D $36(1 + 18)$

- 5 What is the least common multiple (LCM) of any two different prime numbers?
- A the difference between the prime numbers
 - B the product of the prime numbers
 - C the quotient obtained by dividing the greater prime number by the smaller one
 - D the sum of the prime numbers

- 6 At a baseball stadium, every eighth person received a coupon for a free hot dog. Every thirtieth person received a free baseball cap. Which person was the first person to receive both free gifts?

- A 60
- B 120
- C 160
- D 240

- 7 The table below shows the numbers of flowers in bunches that are sold by a flower shop.

**BUNCHES OF FLOWERS SOLD
BY FLOWER SHOP**

Flower	Number in Bunch
Sunflowers	6
African Daisies	8

Bess wants to buy the same number of sunflowers as African daisies. What is the **fewest** number of bunches of each kind of flower she can buy?

- A 3 bunches of sunflowers, 4 bunches of African daisies
- B 4 bunches of sunflowers, 3 bunches of African daisies
- C 6 bunches of sunflowers, 8 bunches of African daisies
- D 8 bunches of sunflowers, 6 bunches of African daisies

- 8 Cindy is having a birthday party, and she wants to make gift bags for her guests. She has 36 game cards and 60 colored pencils to divide among the gift bags. She wants to give the same number of each item to each guest. How many gift bags can she make? How many of each item can go in each bag?

Show your work.

Answer _____ greatest number of gift bags

Answer _____ games cards per bag

Answer _____ colored pencils per bag

- 9 What is the greatest common factor of 20 and 16?

Answer _____

Explain your answer.

EXIT TICKET

NY.6.NS.4

Now that you have mastered least common multiples and greatest common factors, let's solve the problem in the **Real-World Connection**.

Anika and her family are waiting at the airport for a shuttle to take them to their car. They need two shuttles, but both shuttles are leaving as they walk to the shuttle area. One of the shuttles returns to the airport every 9 minutes. The other shuttle returns every 12 minutes. How long will it be before the two shuttles are at the airport at the same time again? Explain your answer.